

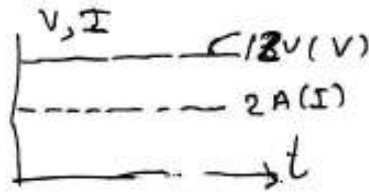
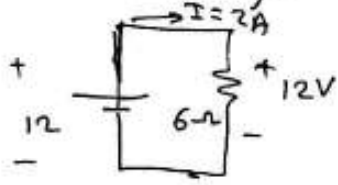
# Lec (01) .. Revision

## Part 1

(AC) alternating current

التيار المتردد

1- Previously, we learned DC sources ↙ dependent  
↘ independent

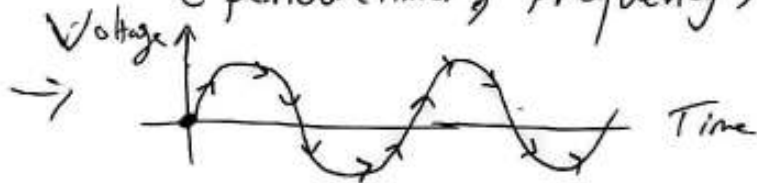


DC source constant values with time (DC = Direct current)

## 2- Ac (alternating current)

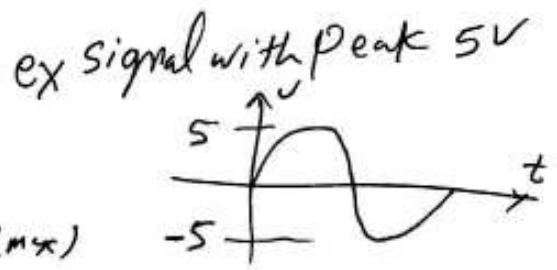
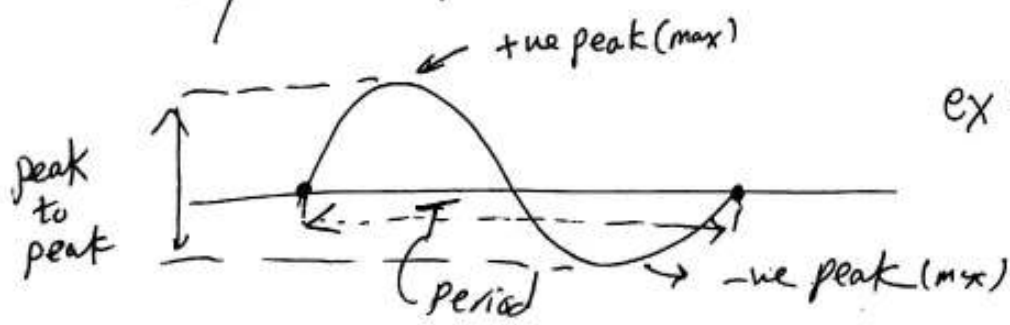
- Voltage & current vary with time in Amplitude & direction
- Sinusoidal wave (Example of AC signals) has some characteristics

(Period (Time), Frequency, relation between period & frequency)

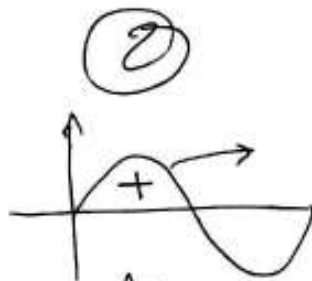


- The Sinusoidal wave starts from (0V), increase to maximum positive value (+ve peak) and then decreased to zero & continue to maximum negative value (-ve peak) & returns to zero again and so on (repeat cycles)
- The waveform (signal) is called periodic because every period it repeats itself.

-- Symbol of AC in Electrical circuit

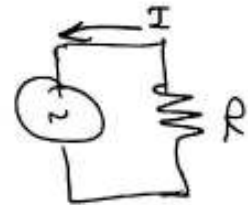
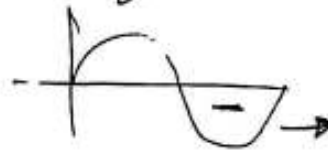


\* Polarity of Sine wave

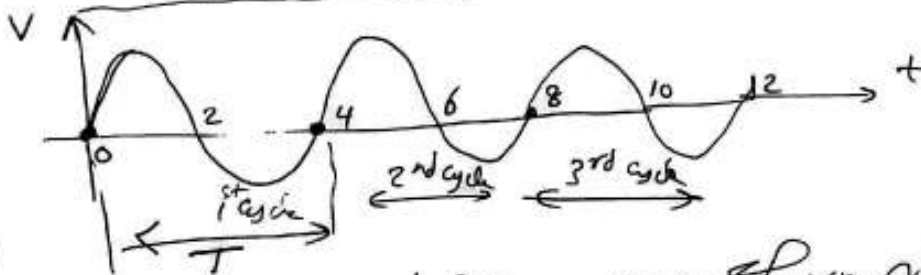


During (+ve half Cycle) → Voltage source is positive & current generated (clockwise)

During (-ve half Cycle) → Voltage source is -ve & current generated (counter clockwise)



\* Period of Sine wave



(EX) Calculate the period of sine wave shown above?

sol  $T = 4 \text{ sec}$

[ 1 cycle = 1 period ]

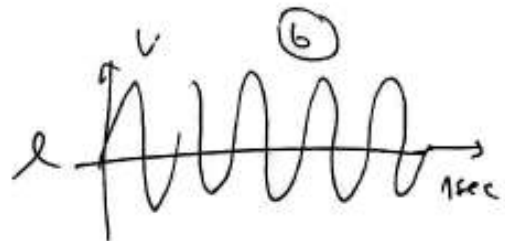
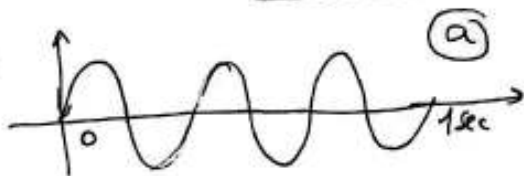
\* Frequency of Sine wave

$f = \frac{1}{T}$

→  $\frac{1}{\text{time}} = \text{Hz}$

$f = \frac{1}{4} = 0.25 \text{ Hz}$

EX(2)



Which of 2 sine waves has more frequency

fig(a) → 3 cycles (sum of time = 1 sec)

∴ one cycle period =  $\frac{1}{3} \text{ sec}$

∴ freq =  $\frac{1}{T} = 3 \text{ Hz}$  or [no. of cycles/sec = 3]

fig(b) → no. of cycles = 5 ∴  $5 \text{ Hz}$  or  $\frac{1}{T} = \frac{1}{\frac{1}{5} \text{ sec}} = 5 \text{ Hz}$

∴ fig (b) more freq. than fig (a)

(3)

EX(3) If the period of certain sine wave is 10ms, what's freq?

Sol  $f = \frac{1}{T} = \frac{1}{10\text{ms}} = \frac{1}{10 \times 10^{-3}} = 100\text{Hz}$

EX(4) The Freq. of sine wave is 60Hz, what's period?

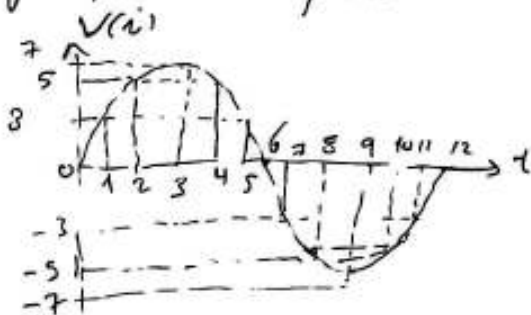
$T = \frac{1}{f} = \frac{1}{60} = 16.7\text{ms}$

Sinusoidal voltage values

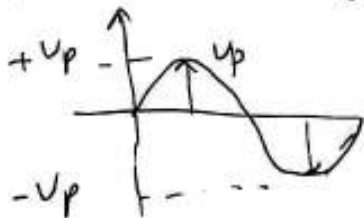
- 1- Find instantaneous value at any time
- 2- find peak
- 3- Peak-to-peak
- 4- RMS
- 5- Average

[1] Instantaneous value

i.e.  $v$  at any time  $t$



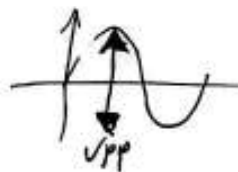
[2] Peak value (positive or negative maximum with respect to zero)



$V_p$

[3] Peak to peak value

$V_{pp} = 2V_p$



[4] RMS (root mean square value)  $\Rightarrow$  AC voltmeter measured value = effective value

مقدار متوسط AC مع مقادير DC عند توصيل الجهد مع مقاومة وينتج عن ذلك حرارة تساوي الحرارة الناتجة عن توصيل الجهد AC مع نفس المقاومة.

$RMS = \frac{V_{max}}{\sqrt{2}}$  ex  $V_{max} = 5\text{V} \therefore V_{RMS} = \frac{5}{\sqrt{2}}, V_{pp} = 10\text{V}$

(4)

[5] Average value (DC Value) (mean Value) = read of DC Avometers  
 = total area under the half cycle wave divided by distance along horizontal axis (For complete cycle = zero)

(in half sine wave)  $\rightarrow V_{avg} = \frac{2V_{max}}{\pi} = \frac{2V_p}{\pi}$

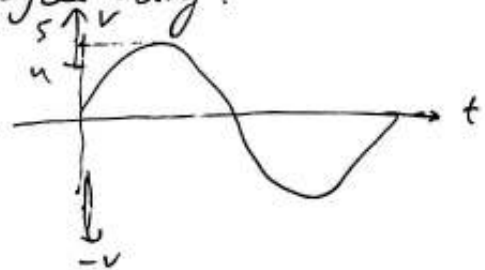
EX(5) find  $V_p$ ,  $V_{pp}$ ,  $V_{rms}$ , The half cycle  $V_{avg}$ .

$V_p = 4.5V$

$V_{pp} = 2 \times V_p = 9V$

$V_{rms} = V_p / \sqrt{2} = \frac{4.5}{\sqrt{2}} = 3.18V$

$V_{avg} = \frac{2V_p}{\pi} = \frac{2}{3.14} \times 4.5 = 2.87V$



angular measurement of sine wave

radian  $\leftrightarrow$  degree

$\frac{\text{degree}}{180^\circ} = \frac{\text{rad}}{\pi}$

So  $1 \text{ degree} = \left(\frac{180^\circ}{\pi}\right) \times \text{rad}$

$1 \text{ rad} = \left(\frac{\pi}{180^\circ}\right) \times \text{degree}$

Example  $180^\circ = ? \text{ rad}$

$\therefore = \frac{\pi}{180} \times 180 = \pi \text{ rad}$

EX(6)

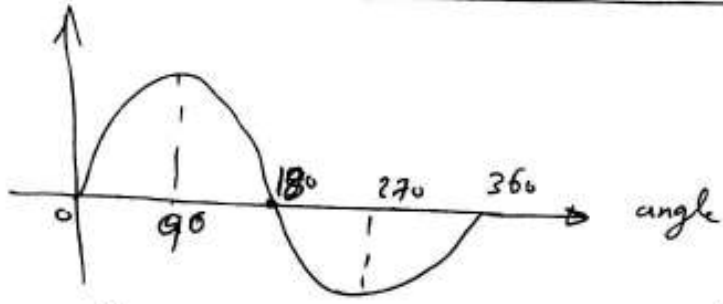
Convert  $60^\circ$  to rad. &  $\frac{\pi}{8}$  rad to degrees

a- Rad =  $\frac{\pi}{180} \times 60 = \pi/3 \text{ rad}$

b- deg  $\Rightarrow \frac{180}{\pi} \times \left(\frac{\pi}{8}\right) = 30^\circ$

Sine wave angles

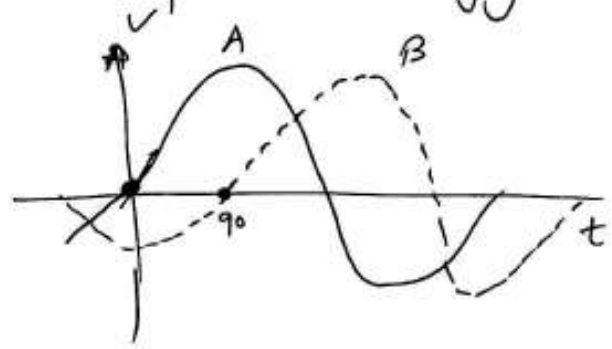
(Phase)



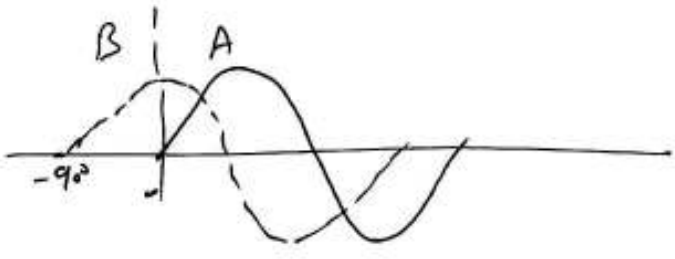
angle  $\Rightarrow$  angular frequency  
 $= \omega t = 2\pi ft \Rightarrow 2\pi$   
 one cycle  $= 2\pi = 360^\circ$

The phase of sine wave is [angular measurement of that specifies the position of that sine wave].

\* it is useful to identify which signals lead / or lag

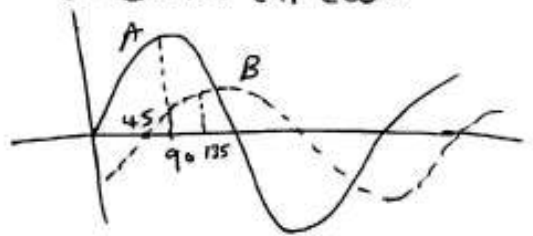


A lead B by  $90^\circ$   
 or B lag A by  $90^\circ$

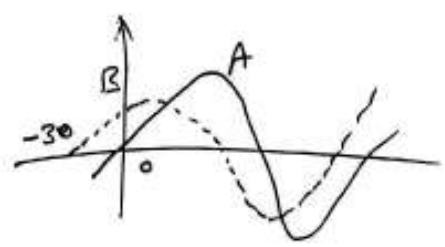


B lead A by  $90^\circ$   
 or A lag B by  $90^\circ$

EX(7) what is the phase angles between the two sine waves in both circuits



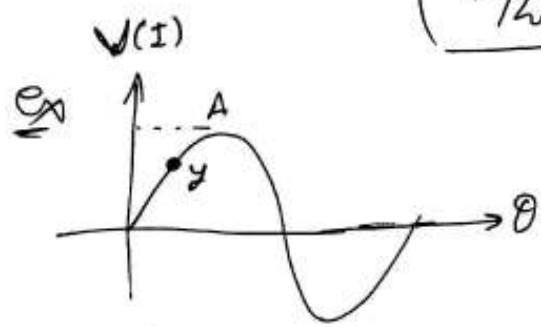
A lead B by  $45^\circ$   
Phase angle  $45^\circ$



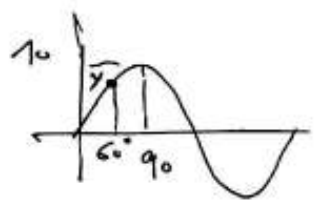
B lead A by  $30^\circ$   
Phase angle  $30^\circ$

6

The Sine wave formula (instantaneous wave)

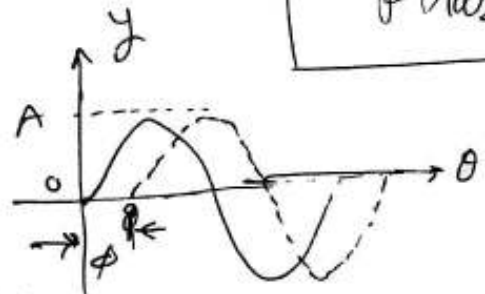


$y = A \sin \theta$   
 instantaneous  
 $V(t) = V_p \sin \theta$

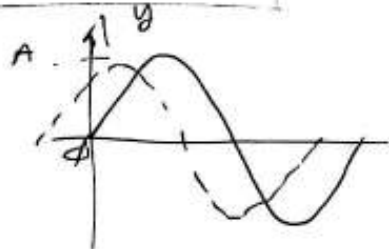


$\rightarrow V = 10 \sin \theta$   
 at  $60^\circ \rightarrow V = 10 \sin 60 = 8.66V$   
 $\therefore y = 8.66V$

Phase shift sine wave

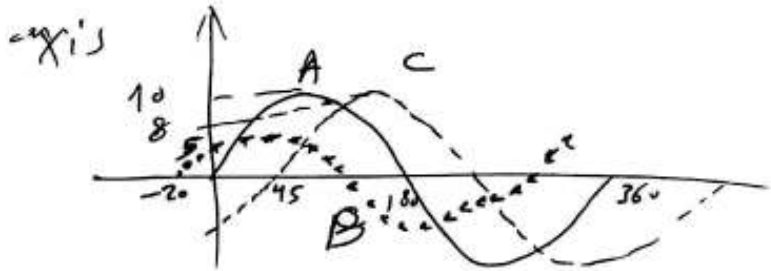


$y = A \sin(\theta - \phi)$   
 reference solid line  
 phase shift



$y = A \sin(\theta + \phi)$   
 reference solid line  
 phase shift

EX 18) Determine instantaneous value at  $90^\circ$  reference point on horizontal



$V_A = A \sin \theta = 10 \sin \theta \Big|_{90^\circ} = 10 \sin 90 = 10V$   
 (ref)

$V_B = 5 \sin(\theta + 20) \Big|_{\theta=90} = 5 \sin(110) = 4.7V$   
 phase shift

$V_C = 8 \sin(\theta - 45) = 8 \sin(90 - 45) = 5.66V$   
 phase shift



1] Capacitors

symbol

two plates charged with ac source

Capacitance =  $C = \frac{Q \rightarrow \text{charge}}{V \rightarrow \text{applied voltage}} = \frac{\epsilon_0 \epsilon_r A}{d}$

or  $Q = CV$

its unit is Farad (F)  $\rightarrow$  actually (micro, nano, Pico Farad)

$\epsilon_0 \epsilon_r A$  labels:  $8.85 \times 10^{-12} \text{ F/m}$  (permittivity),  $A$  (area of plates),  $d$  (dist. betw plates)

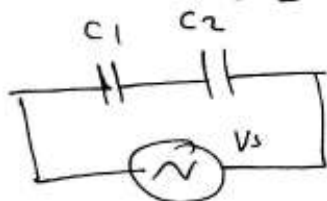
EX(1) A certain capacitor stores 50 microcoulombs with 10V across its plates. What is capacitance (in microfarad)

sol/  $C = Q/V = \frac{50 \text{ micro}}{10} = 5 \mu\text{F}$

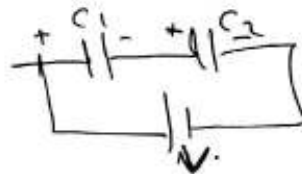
EX(2) determine the capacitance of parallel plates having area  $0.01 \text{ m}^2$  & separation  $0.5 \text{ mil} = [1.27 \times 10^{-5} \text{ m}]$ , Dielectric is mica (has  $\epsilon_r$  of 5)

sol  $C = \frac{\epsilon_0 \epsilon_r A}{d} = \frac{(8.85 \times 10^{-12})(5)(0.01)}{1.27 \times 10^{-5}} = 0.035 \mu\text{F}$

a- Series connection of capacitors



or



note  $Q_1 = Q_2 = Q_3 = \dots$   
 $\therefore C_T = \frac{C}{n}$

$V_T = V_1 + V_2 + \dots$

$\frac{Q_T}{C_T} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2} + \dots$

$\therefore \frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \dots$

$Q_1 = Q_2 = Q_3 = \dots$

series =  $\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \dots$   
 Parallel =  $C_T = C_1 + C_2 + \dots$

Series Capacitor Like Parallel Resistors

EX(3) Find total capacitance between A & B

$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \quad \therefore \frac{1}{C_T} = \frac{1}{2.3 \mu\text{F}} \quad \therefore C_T = 2.3 \mu\text{F}$

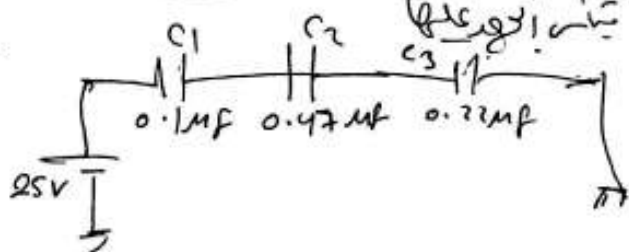
2

note if you want to use voltage divider in series connection to determine voltage across each individual capacitor, So:-

$$V_1 = V_T \times \frac{X_{C1}}{X_{CT}} = \left( V_T \cdot \frac{C_T}{C_1} \right) \rightarrow \text{total cap}$$

EX(4) find voltage across each capacitor

$$C_T = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}} = 0.06 \mu\text{F}$$



$$V_1 = V_T \frac{C_T}{C_1} = 25 \times \frac{0.06 \mu\text{F}}{0.1 \mu\text{F}} = 15\text{V}$$

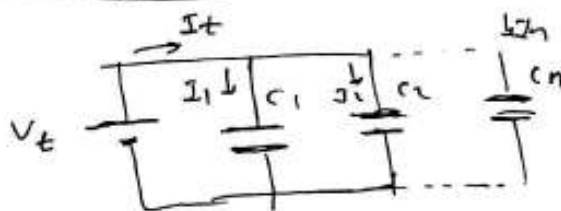
$$V_2 = V_T \frac{C_T}{C_2} = 25 \times \frac{0.06 \mu\text{F}}{0.47 \mu\text{F}} = 3.19\text{V}$$

$$V_3 = V_T \frac{C_T}{C_3} = 25 \times \frac{0.06 \mu\text{F}}{0.22 \mu\text{F}} = 6.81\text{V}$$

### b- Parallel Connection of C

$$Q_T = Q_1 + Q_2 + Q_3 + \dots + Q_n$$

$$C_T V_T = C_1 V_1 + C_2 V_2 + C_3 V_3 + \dots + C_n V_n$$

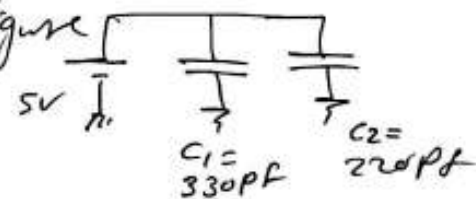


but  $V_T = V_1 = V_2 = \dots = V_n$

$$\therefore C_T = C_1 + C_2 + C_3 + \dots + C_n$$

So Parallel C  $\equiv$  Series R

EX(5) What is the voltage across each cap of figure



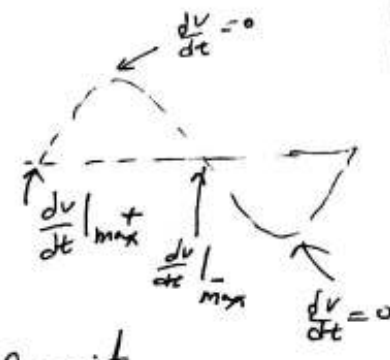
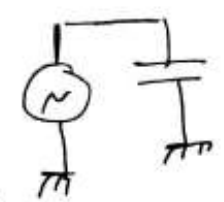
$$C_T = C_1 + C_2 = 330 + 220 = 550 \text{ pF}$$

$$V_T = V_1 = V_2 = 5\text{V}$$



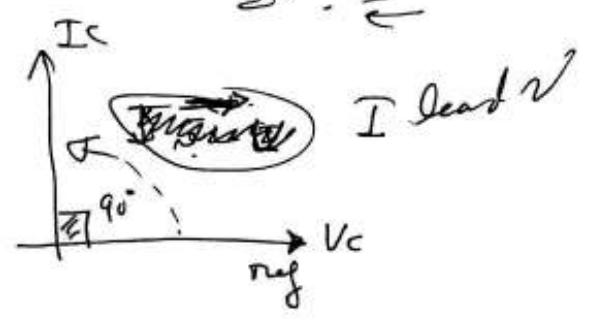
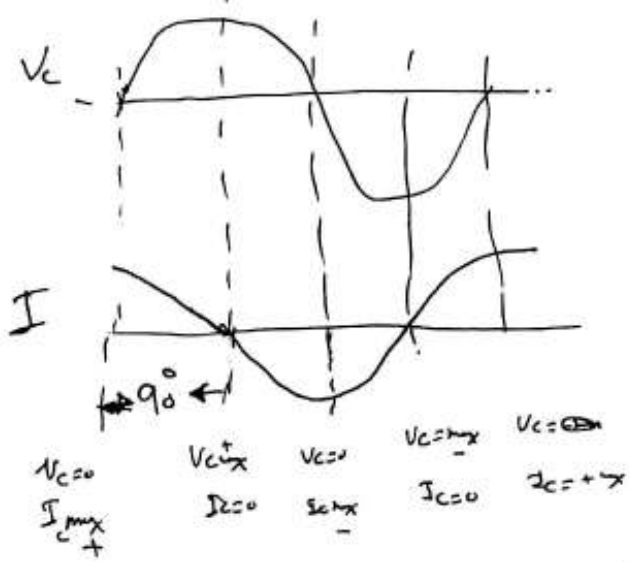
(3)

in capacitor  
 $i = c \frac{dv}{dt}$



Relation between I & V in capacitor

**I** lead **V** by  $90^\circ$  in pure capacitive circuit

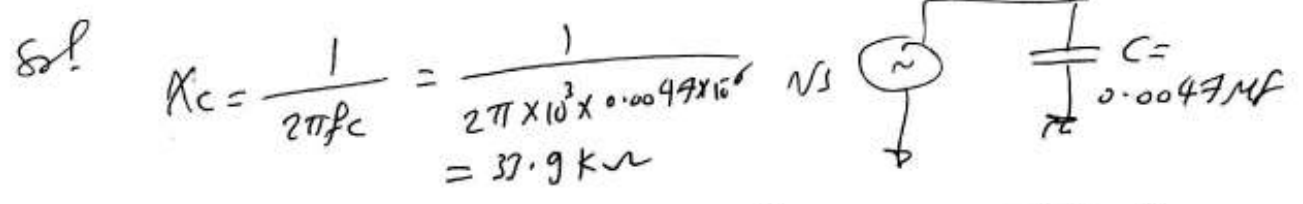


Capacitive Reactance

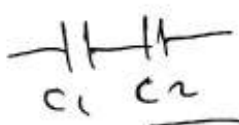
$X_c = \frac{1}{2\pi f c}$   
 magnitude

also written  
 $-jX_c$  or  $\frac{1}{jX_c}$   
 phase  $(-90^\circ)$

Ex(6) For the circuit shown, determine capacitive reactance of  $f = 1\text{kHz}$



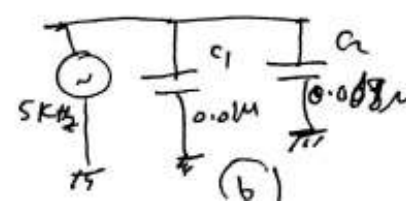
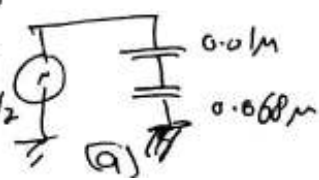
(a)  $X_{CT} = X_{C1} + X_{C2}$



$X_{CT} = X_{C1} + X_{C2}$   
 $X_{CT} = \frac{X_{C1} \cdot X_{C2}}{X_{C1} + X_{C2}}$

Ex(7) Calc. total capacitive reactance for  $\omega^t$ s

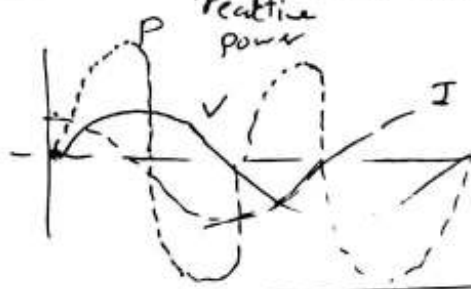
(a)  $X_{CT} = X_{C1} + X_{C2} = \frac{1}{2\pi \times 5\text{K} \times 0.01\mu} + \frac{1}{2\pi \times 5\text{K} \times 0.068\mu}$   
 $= 305 \text{ K}\Omega$   
 (b)  $X_{CT} = \frac{X_{C1} X_{C2}}{X_{C1} + X_{C2}} = 408 \mu$



(4)

Power in capacitor

$$P = V_{rms} I_{rms} = \frac{V_{rms}^2}{X_c} = I_{rms}^2 X_c$$



→ True Power = 0 (ideal capacitor)  
يعني

Note:- Power supply filters also capacitor  
يعني

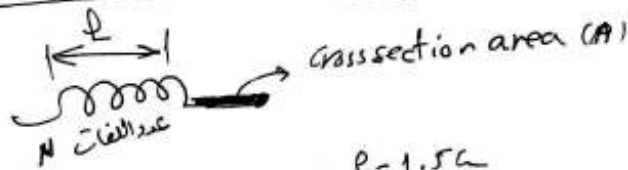
2- Inductors (coils)

coil with (N) turns

the induced voltage  $(V_{ind} = L \frac{di}{dt})$

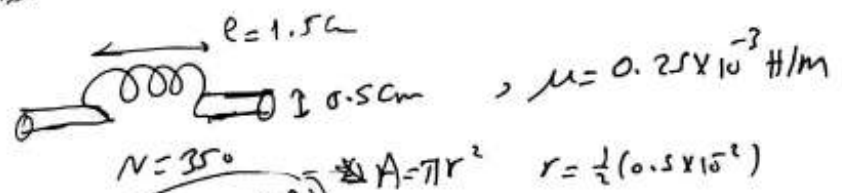
inductance (henry)  
current rate (Amp/sec)

$$L = \frac{N^2 \mu A}{l}$$



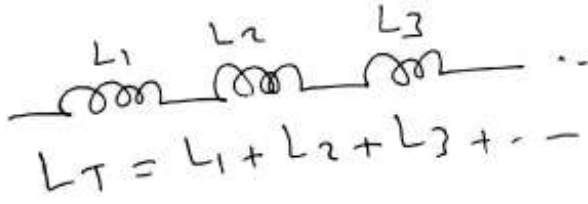
يعني  $A = \pi r^2$   
مساحة القطر

$\Sigma X$



$$L = \frac{(350)^2 \times (0.25 \times 10^{-3}) \times (\pi (0.25 \times 10^{-2})^2)}{1.5 \times 10^{-2}} = 40mH$$

Series connection



$$L_T = L_1 + L_2 + L_3 + \dots$$

سلسلة (سلسلة)

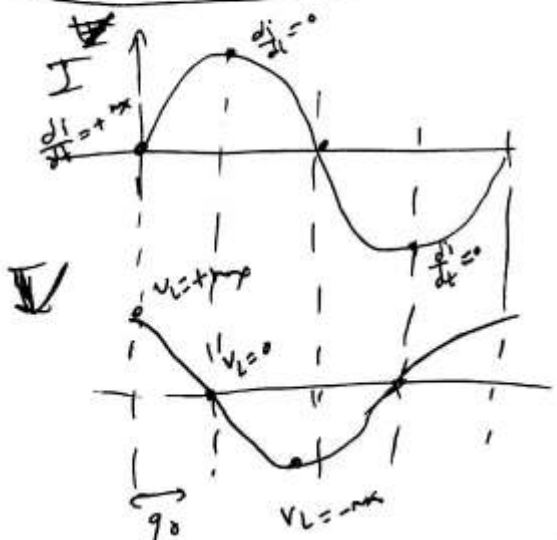
Parallel connection



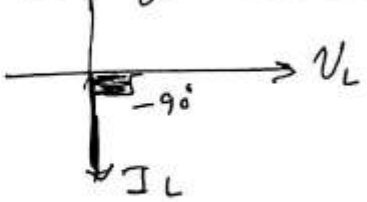
$$\frac{1}{L_T} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \dots$$

متوازية (متوازية)

Relation between I & V in inductor



I lag V by 90°

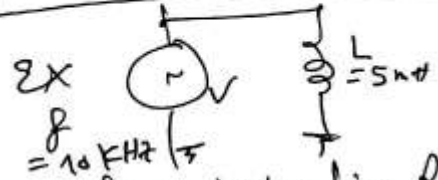


inductive reactance

$$X_L = 2\pi fL$$

مقاومت القابلية، الخرجية

$$jX_L = j2\pi fL = j\omega L$$



find inductive reactance

Sol:  $X_L = 2\pi fL = 2\pi \times 10 \times 10^3 \times (5 \times 10^{-3}) = 314 \Omega$

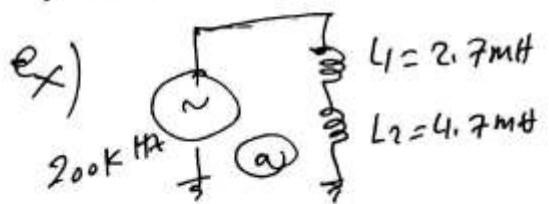


$$X_{LT} = X_{L1} + X_{L2} + \dots$$



$$\frac{1}{X_{LT}} = \frac{1}{X_{L1}} + \frac{1}{X_{L2}} + \dots$$

مقاومت القابلية  
مقاومت القابلية  
مقاومت القابلية  
مقاومت القابلية



$$X_{L1} = 2\pi fL_1 = 2\pi \times (200 \times 10^3) \times (2.7 \times 10^{-3}) = 3.39 \text{ k}\Omega$$

$$X_{L2} = 2\pi fL_2 = 2\pi \times (200 \times 10^3) \times (4.7 \times 10^{-3}) = 5.91 \text{ k}\Omega$$

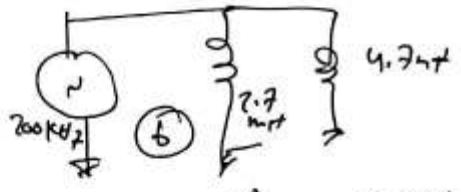


Fig (a)  $X_{L \text{ tot}} = X_{L1} + X_{L2} = 9.3 \text{ k}\Omega$

Fig (b)  $X_{L \text{ tot}} = \frac{X_{L1} \cdot X_{L2}}{X_{L1} + X_{L2}} = 2.15 \text{ k}\Omega$

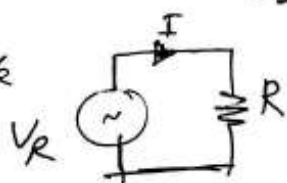
Power in inductor

$$P_{\text{react}} = V_{\text{rms}} I_{\text{rms}} = \frac{V_{\text{rms}}^2}{X_L} = I_{\text{rms}}^2 X_L$$

# ⑥ R, L, C in Circuits

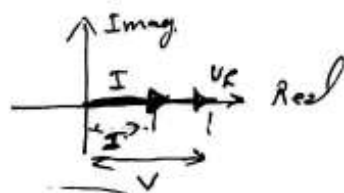
- ① Pure Resistance    ② Pure capacitance    ③ Pure inductance  
 ④ series RL    ⑤ Series RC    ⑥ series RLC  
 ⑦ General Case    ⑧ Problem    ⑨ Admittance

## ① Pure Resistance

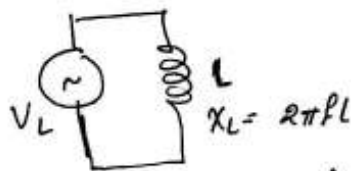
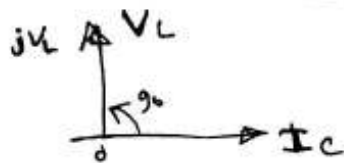


impedance  $Z = \frac{V_R}{I} = \frac{V_R L_0}{I_R L_0} = R L_0$

note:  $V, I$  in the same phase in pure resistance



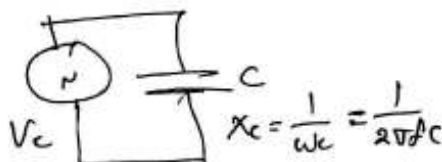
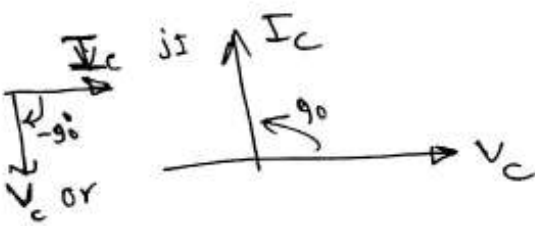
## ② Pure inductance



$Z = \frac{V_L L_0}{I_L L_0} = \frac{X_L L_0}{1} = jX_L = j\omega L = j2\pi fL$

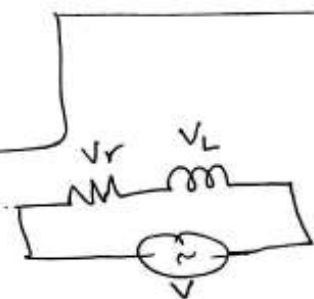
$I \text{ lag } V$

## ③ Pure capacitance

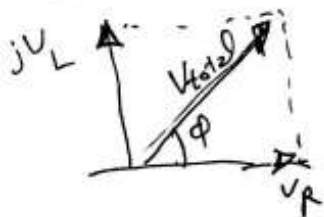


$Z = \frac{V_C L_0}{I_C L_0} = \frac{X_C L_0}{1} = -jX_C = -\frac{j}{\omega C} = -\frac{j}{2\pi fC}$

## ④ series RL



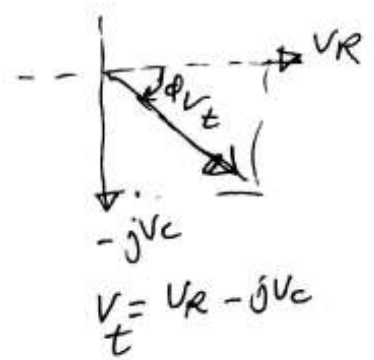
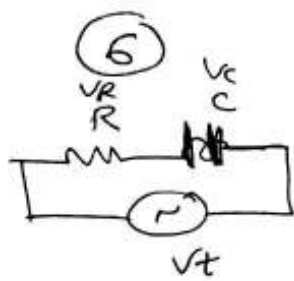
The phasor is sum of  $V_R$  &  $V_L$   
 Then  $I$  lag  $V$  by angle between  $(0, 90^\circ)$



$Z = 3 + j4 = 5 \left[ \tan^{-1} \frac{4}{3} \right]$   
 R     $X_L = 2\pi fL$     magnitude 5    phase ( $\phi$ )

5 - series RC

I lead V by angle between (0 to 90°)



$$Z = R - jX_c = R - j\left(\frac{1}{\omega C}\right)$$

$$= \left[ R + X_c = R + \frac{1}{j\omega C} = R - j\left(\frac{1}{\omega C}\right) \right]$$

magnitude =  $\sqrt{R^2 + X_c^2}$  , phase  $\tan^{-1}\left(-\frac{X_c}{R}\right)$

6 - Series RLC



$$V = V_R + j(V_L - V_C)$$

$$Z = R + j(X_L - X_C)$$

$$= \sqrt{R^2 + (X_L - X_C)^2} \left[ \tan^{-1}\left(\frac{X_L - X_C}{R}\right) \right]$$

note This circuit used in (Resonance circuit)

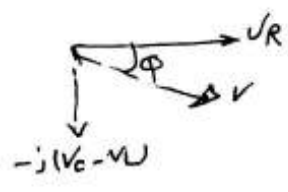


There are 3 cases

1- if  $X_L = X_C$  ( $V_L = V_C$ )  $\therefore Z = R$  (pure resistance)  
 (resonance)  $\omega L = \frac{1}{\omega C}$

2- if  $X_L > X_C$  ( $V_L > V_C$ )  $\therefore Z = R + j(X_L - X_C)$   
 circuit is inductive effect

3- if  $X_L < X_C$  ( $V_L < V_C$ )  $\therefore Z = R + j(X_L - X_C)$   
 circuit is capacitive effect



# Part (3)

1

## AC Circuit Theory Techniques

Node  
Mesh  
Delta-Y  
Source transformation  
Superposition  
Thevenin's & Norton's

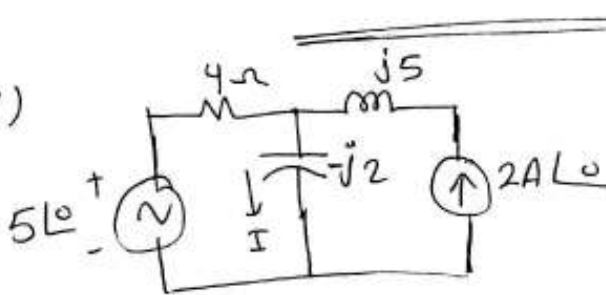
حل دوائر التيار المتردد AC  
Components  
Source

1 Don't forget  $Z = \frac{V}{I} = \text{magnitude} \angle \text{phase}$

حل

Power factor =  $\cos[\text{phase}]$   
 PF  $\angle$  Z  $\angle$   $\cos$  (زاوية)

EX(1)



Determine I using

- \* - Node
- \* - Mesh
- \* - superposition
- \* - Thevenin's
- \* - source transformation

### 1 Node method

هذا هو

$$\infty \frac{N_1 - 5\angle 0^\circ}{4} + \frac{N_1}{-j2} + 2A\angle 0^\circ = 0$$

$$\infty \frac{V_1}{4} - \frac{5}{4} - \left[ \frac{V_1}{2j} \right] - 2 = 0$$

$$\frac{V_1}{4} - \frac{5}{4} - \left[ \frac{V_1}{2j} - \frac{j}{j} \right] - 2 = 0$$

$$\rightarrow \frac{V_1}{4} - \frac{5}{4} + \frac{jV_1}{2} - 2 = 0$$

$$N_1 \left[ \frac{1}{4} + \frac{j}{2} \right] = \frac{5}{4} + 2 = \frac{13}{4}$$

$$V_1 = \frac{13/4}{\frac{1}{4} + j/2} = \frac{13}{1 + 2j} = \frac{13\angle 0^\circ}{\sqrt{1+4} \angle \tan^{-1} 2} = \frac{13\angle 0^\circ}{2.23 \angle 63.43^\circ}$$

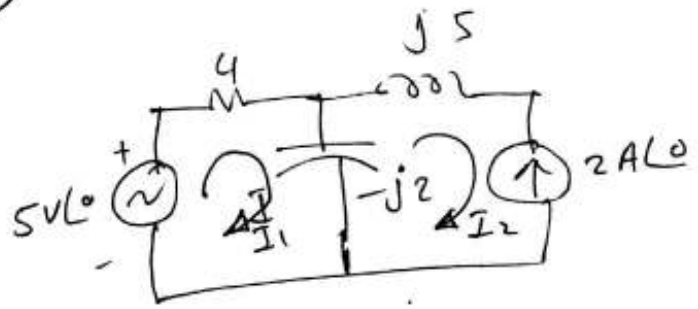
$$V_1 = 5.81 \angle -63.43^\circ$$

$$\infty I = \frac{V_1}{-j2} = \frac{5.81 \angle -63.43^\circ}{2 \angle -90^\circ} = 2.9 \angle 26.56^\circ \text{ A}$$



(2)

## 2- Mesh Poop



loop 1

$$5 \angle 0^\circ = (4 - j2)I_1 - (-j2I_2)$$

$$\therefore 5 \angle 0^\circ = (4 - j2)I_1 + 2jI_2 \rightarrow (1)$$

loop 2

$$I_2 = -2 \angle 0^\circ$$

من محتاج معادله

substitute in (1)  $\therefore 5 = (4 - j2)I_1 + 2j \times (-2)$

$$5 = [4 - 2j]I_1 - 4j \rightarrow (5 + 4j) = (4 - 2j)I_1$$

$$\therefore I_1 = \frac{5 + 4j}{4 - 2j} = \frac{\sqrt{25+16} \angle \tan^{-1} \frac{4}{5}}{\sqrt{16+4} \angle \tan^{-1} \frac{-2}{4}} = \frac{6.4 \angle 38.66^\circ}{4.47 \angle -26.56^\circ}$$

$$I_1 = 1.43 \angle 65.22^\circ$$

∴  $I = I_1 - I_2 = 1.43 \angle 65.22^\circ - (-2)$

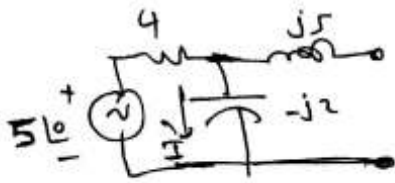
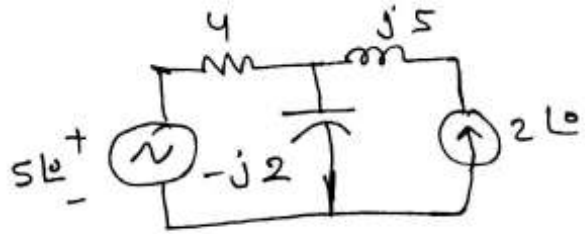
$$= (1.43 \cos 65.22^\circ) + j(1.43 \sin 65.22^\circ) + 2$$

$$I = 2.6 + 1.3j = 2.9 \angle 26.56^\circ$$

### ③ Superposition

[3].

① 1<sup>st</sup> step open circuit current source



ثم نطبق اى نظرية قدرية لحساب التيار  $I'$  ونسويه  $I'$

∴ open circuit  $2\angle 0^\circ$ .

الدائرة تكافئ دائرة مقفلة

$$\therefore I' = \frac{5\angle 0^\circ}{4 - 2j} = \frac{5\angle 0^\circ}{4.47 \angle -26.57^\circ} = 1.118 \angle 26.57^\circ$$

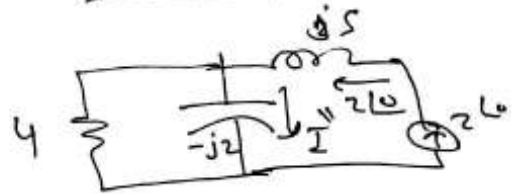
② 2<sup>nd</sup> step S.C voltage source



using current divider to calc.  $I''$

$$I'' = \frac{I_{\text{total}} \times 4}{4 - 2j} = \frac{2 \times 4\angle 0^\circ}{4 - 2j}$$

$$= \frac{8\angle 0^\circ}{4.47 \angle -26.57^\circ} = 1.789 \angle 26.57^\circ$$

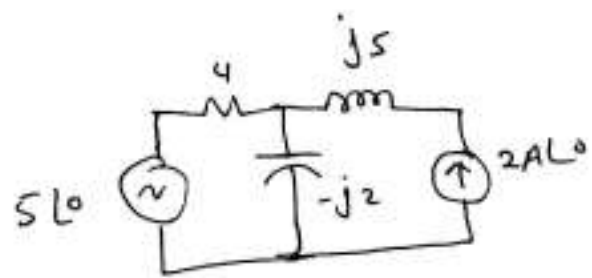


$$\therefore I = I' + I'' = 1.118 \angle 26.57^\circ + 1.789 \angle 26.57^\circ$$

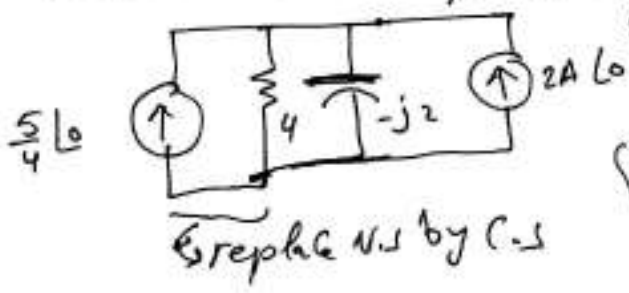
$$= 2.9 \angle 26.56^\circ$$

(4)

4) Source transformation

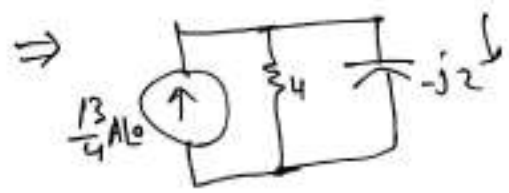


1<sup>st</sup> equivalent circuit



Coil removed has no effect with independent source  
 series independent voltage source and independent current source  
 element  $j5$  is element in series  
 o.c. element in series independent voltage source

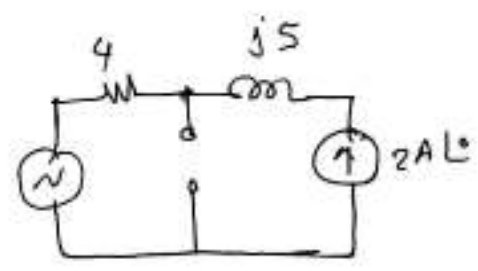
Graph  $V_{th}$  by C.S



$$I = \frac{\frac{13}{4} \angle 0 \times 4}{4 - j2} = \frac{13 \angle 0}{4.47 \angle -26.56^\circ} = 2.9 \angle 26.56^\circ$$

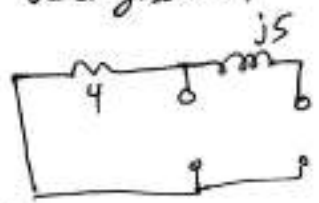
5) Thouvenin's 1) open circuit the capacitor

2) calc.  $V_{th}$ ,  $Z_{th}$  across cap (جهد و المقاومة)



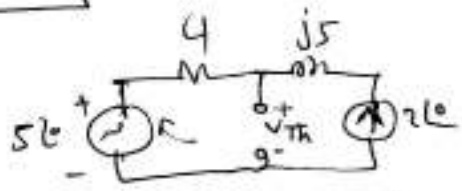
a) to calc  $Z_{th}$  (o.c current source, s.c voltage source)

j5 cancelled open -ve



$Z_{th} = 4 \Omega$  only

b) calc.  $V_{th}$  using KVL

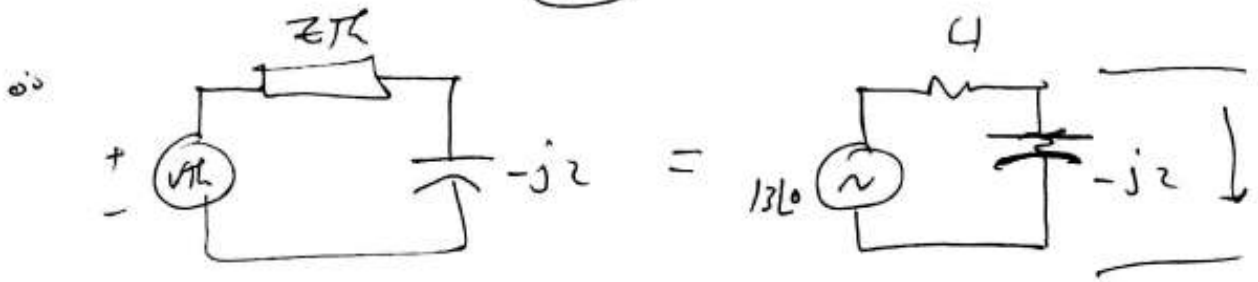


loop 1  $5 - V_{th} = 4I$

loop 2  $V_{th} = 2 \times 2$   $I = -2 \angle 0$

$\therefore 5 - V_{th} = 4 \times -2$   
 $V_{th} = 13 \angle 0$

(5)



$$I = \frac{13\angle 0}{4 - j2} = 2.9 \angle 26.56$$

لا يمكن استعمال  $\Delta$ -Y، بل يجب لأنه لا استمرارية في قيمه فكانت وقتية  
المكثف المراد حساب التيارية

مفصل للرجوع  $\Delta$  و  $Y$  المحاضرة القادمة  
DC & AC

Good Luck!

Quiz

$$2\angle 30 + j2\sin 30$$

لا حظ لو قابليت معادله (Meshloop) في

$$2\angle 30 = (2 + j)I_1 + (3 - 2j)I_2$$

$$4\angle 60 + j4\sin 60$$

$$4\angle 60 = (5 - 2j)I_1 + (4)I_2$$

تاكيد انا !! الا انهم يتكلمون بالبرهان،  $\Delta$  و  $Y$  في  $\Delta$  و  $Y$  في  $\Delta$  و  $Y$  في  $\Delta$

$$\Delta = \begin{vmatrix} 2+j & 3-2j \\ 5-2j & 4 \end{vmatrix} = \text{نقطة} \quad \Delta_1 = \begin{vmatrix} 2\angle 30 + j2\sin 30 & 3-2j \\ 4\angle 60 + j4\sin 60 & 4 \end{vmatrix} = \text{نقطة}$$

$$\Delta_2 = \begin{vmatrix} 2+j & 2\angle 30 + j2\sin 30 \\ 5-2j & 4\angle 60 + j4\sin 60 \end{vmatrix} = \text{نقطة}$$

$$\therefore I_1 = \frac{\Delta_1}{\Delta} \quad I_2 = \frac{\Delta_2}{\Delta}$$